OCR Additional Maths Exam Questions -Factor and Remainder Theorems

14 **(a)** **(i)** On the same graph, draw sketches of the curve $y = x^3$ and the line $y = 3 - 2x$. **[2]**

(ii) Use your sketch to explain why the equation $x^3 + 2x - 3 = 0$ has only one root. **[1]**

(b) **(i)** Show by differentiation that there are no stationary points on the curve $y = x^3 + 3x - 4$. **[3]**

(ii) Hence explain why the equation $x^3 + 3x - 4 = 0$ has only one root. **[1]**

(c) **(i)** Use the factor theorem to find an integer root of the equation $x^3 + x - 10 = 0$. **[1]**

(ii) Write the equation $x^3 + x - 10 = 0$ in the form $(x - a)(x^2 + px + q) = 0$ where a, p and q are values to be determined. **[2]**

(iii) By considering the quadratic equation $x^2 + px + q = 0$ found in part **(ii)**, show that the cubic equation $x^3 + x - 10 = 0$ has only one root. **[1]**

(d) You are given that r and s are positive numbers. What do the results in parts **(a)**, **(b)** and **(c)** suggest about the equation $x^3 + rx - s = 0$? **[1]**

3 The function $f(x)$ is defined by $f(x) = x^3 - 5x^2 + 2x + 8$.

(i) Find the remainder when $f(x)$ is divided by $(x + 1)$. **[2]**

(ii) Solve the equation $f(x) = 0$. **[3]**

7 **(a)** Determine whether or not each of the following is a factor of the expression $x^3 - 7x + 6$. You must show your working.

(i) $(x - 2)$ **[2]**

(ii) $(x + 1)$ **[1]**

(b) **(i)** Factorise the function $f(x) = x^3 - 7x + 6$. **[3]**

(ii) Solve the equation $f(x) = 0$. **[1]**

3 The function $f(x) = x^3 + ax + 6$ is such that when $f(x)$ is divided by $(x - 3)$ the remainder is 12.

(i) Show that the value of a is -7. **[2]**

(ii) Factorise $f(x)$. **[3]**

6 The function $f(x) = x^3 - 4x^2 + ax + b$ is such that

- $x = 3$ is a root of the equation $f(x) = 0$,
- when $f(x)$ is divided by $(x - 1)$ there is a remainder of 4.

(i) Find the value of a and the value of b. **[4]**

(ii) Solve the equation $f(x) = 0$. **[3]**

8 The cubic polynomial $f(x) = x^3 + ax + 6$, where a is a constant, has a factor of $(x + 3)$.

(i) Find the value of a. **[2]**

(ii) Hence or otherwise, solve the equation $f(x) = 0$ for this value of a. **[4]**

2 The function $f(x)$ is defined by $f(x) = x^3 - 4x^2 + 5x - 2$.

(i) Find the remainder when $f(x)$ is divided by $(x + 2)$. [2]

(ii) Show that $(x - 1)$ is a factor of $f(x)$. [1]

(iii) Hence solve the equation $f(x) = 0$. [4]

9 You are given that $f(x) = x^3 - 4x^2 + x + 6$.

(i) Find the remainder when $f(x)$ is divided by $(x - 1)$. [1]

(ii) Show that $(x - 3)$ is a factor of $f(x)$. [2]

(iii) Hence solve the equation $f(x) = 0$. [4]

11 **(a)** You are given that $f(x) = x^3 - 3x^2 - 4x$.

(i) Find the three points where the curve $y = f(x)$ cuts the x-axis. [4]

(ii) Sketch the graph of $y = f(x)$. [1]

(b) You are given that $g(x) = x^3 - 3x^2 - 4x + 12$.

(i) Find the remainder when $g(x)$ is divided by $(x + 1)$. [2]

(ii) Show that $(x - 2)$ is a factor of $g(x)$. [1]

(iii) Hence solve the equation $g(x) = 0$. [4]

9 The cubic equation $x^3 + ax^2 + bx - 26 = 0$ has 3 positive, distinct, integer roots.

Find the values of a and b. [5]